# DECOMPOSING A MULTI-CONTROLLED TOFFOLI GATE

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## 1 Solution

We start with the review of gates provided by Qiskit `QuantumCircuit` class. We notice that the largest relative phase Toffoli-like gate is `RC3XGate()`. We can use this gate to compute *and* value of three out of fourteen qubits and assign the value into one of the ancillas. Since we are left with one spare control (we have 14 qubits in the problem statement, whereas five available ancillas can provide us with 15 spots), we can use it to already include one of the ancillas!

In other words, we need to execute something like this:

```
qc.rcccx(ctrl[0], ctrl[1], ctrl[2], anc[0])
qc.rcccx(ctrl[3], ctrl[4], ctrl[5], anc[1])
qc.rcccx(ctrl[6], ctrl[7], ctrl[8], anc[2])
qc.rcccx(ctrl[9], ctrl[10], ctrl[11], anc[3])
qc.rcccx(ctrl[12], ctrl[13], anc[0], anc[4])
```

And now we only need to apply the standard CCCCX gate to compute the total and-value of the states stored in ancillas 1, 2, 3 and 4, and put the answer in the target qubit. In qiskit, this is done with `qc.mcx([anc[1], anc[2], anc[3], anc[4]], tar)`. Let's not forget to uncompute the ancillas.

We execute standard optimization techniques from qiskit library (ie. 1Q Gate Optimization and CX Cancellation) and we obtain the circuit with **depth 101**.

## 2 Validation

Since the full unitary is rather large, we are satified with testing, whether no phase and no amplitude is changed (beside the expected X gate application for all-ones state). To do that, we execute two scenarios: whether the target qubit is in the state $|0\rangle$ or $|1\rangle$.

We prepare the control state into $|+\rangle^{14}$ and target qubit into the state $|f\rangle$ for each $f \in \{0, 1\}$. Now, after applying the tested decomposition, we only have to check, whether no amplitude and no phase was changed, which can be very easily done with the Statevector Simulator.

During the preparation of the validator, we also count the number of gates and the circuit depth.